

Nuclear modification of di-jet momentum imbalance in p+Pb collisions

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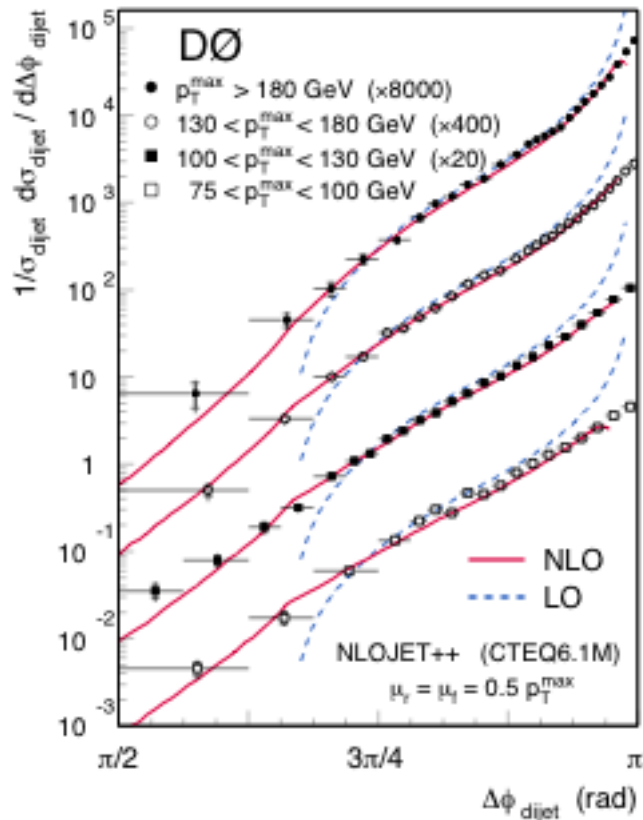
Contents

- Motivations
 - nuclear modification in p+A of the *transverse* structure of di-jets
 - the interesting properties of QCD one can study
 - need to look in the forward rapidity region
- Theory of forward di-jet production in p+A collisions
 - sensitivity to transverse-momentum-dependent (TMD) parton distributions of nuclei at small x
- Phenomenology in p+Pb collisions at the LHC
 - some predictions to motivate a measurement

Motivations

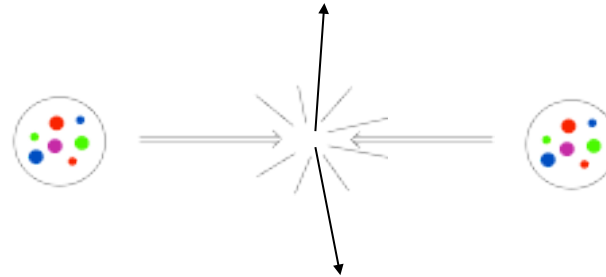
Di-jets in perturbative QCD

in standard pQCD calculations, di-jets are nearly back-to-back



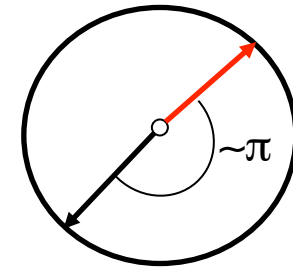
this is supported by hadron collider data
(Tevatron and LHC)

longitudinal view



longitudinal imbalance
almost arbitrary

transverse view

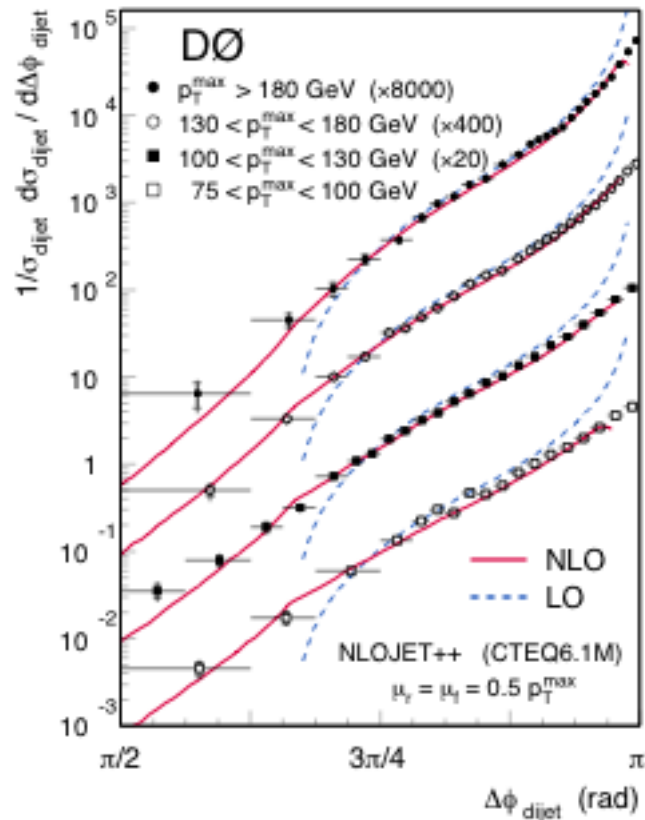


very small
transverse imbalance

peak narrower with higher p_T

Di-jets in perturbative QCD

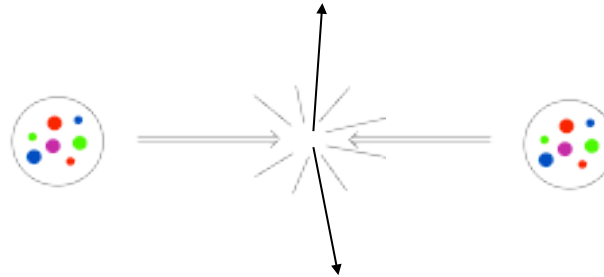
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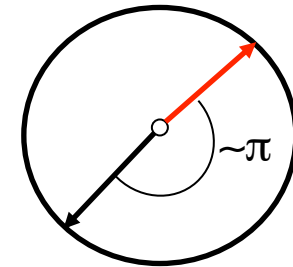
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longitudinal view



longitudinal imbalance
almost arbitrary

transverse view



very small
transverse imbalance

the transverse imbalance is due to 3-jet
(or more) final states, since the incoming
parton transverse momentum is zero
in collinear factorization

Collinear factorization

in standard pQCD calculations, the incoming parton transverse momenta are set to zero in the matrix element and are integrated over in the parton densities

$$d\sigma_{AB \rightarrow X} = \sum_{ij} \int dx_1 dx_2 \underbrace{f_{i/A}(x_1, \mu^2) f_{j/B}(x_2, \mu'^2)}_{\text{k}_\perp \text{ integrated quantities}} d\hat{\sigma}_{ij \rightarrow X} + \mathcal{O}(\Lambda_{QCD}^2/M^2)$$

↓

the incoming partons
are taken collinear to
the projectile hadrons

↓

some
hard scale

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↓ ↓

the incoming partons are taken collinear to the projectile hadrons some hard scale

in the case of di-jets final states: (p_{t1}, y_1) and (p_{t2}, y_2)

$$x_{1,2} = \frac{1}{\sqrt{s}} (|p_{t1}| e^{\pm y_1} + |p_{t2}| e^{\pm y_2}) \quad p_{t1} + p_{t2} = 0$$

nPDF framework: good to describe nuclear modifications of the longitudinal distribution of di-jets

CMS paper 1401.4433

however, transverse-momentum-dependent (TMD) factorization is needed to describe nuclear modifications of the transverse imbalance of di-jets

TMD factorization

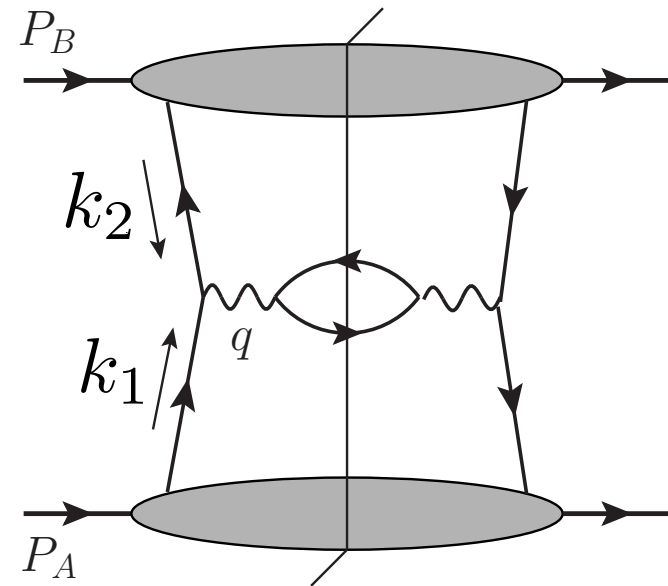
this is a more advanced QCD factorization framework
which can be useful and sometimes is even necessary

the transverse momentum of the di-jet
system q_T is the sum of the transverse
momenta of the incoming partons

$$d\hat{\sigma} \propto \delta(k_{1t} + k_{2t} - q_T)$$

so in collinear factorization

$$d\sigma^{AB \rightarrow J_1 J_2 X} \propto \delta(q_T) + \mathcal{O}(\alpha_s)$$



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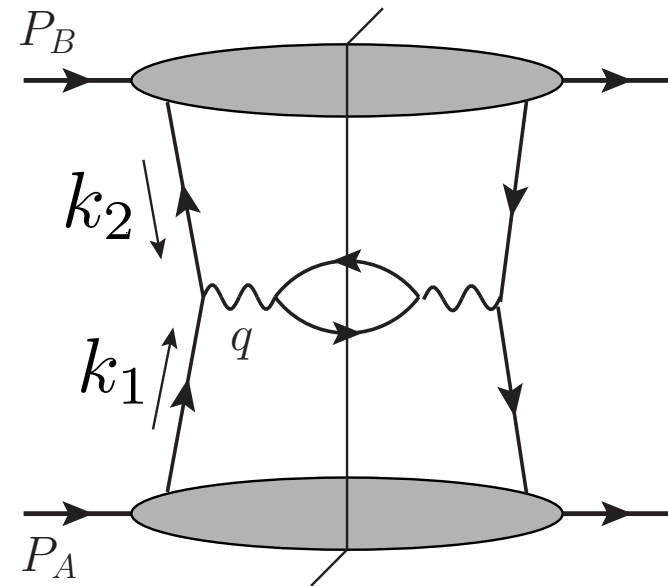
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$$d\sigma^{AB \rightarrow J_1 J_2 X} \propto \delta(q_T) + \mathcal{O}(\alpha_s)$$

naively, TMD factorization is

$$d\sigma^{AB \rightarrow J_1 J_2 X} = \sum_{i,j} \int dx_1 dx_2 d^2 k_{1t} d^2 k_{2t} f_{i/A}(x_1, k_{1t}) f_{j/B}(x_2, k_{2t}) d\hat{\sigma}^{ij \rightarrow J_1 J_2}$$

but unfortunately, this is not so simple



TMD gluon distributions

- the naive operator definition is not gauge-invariant

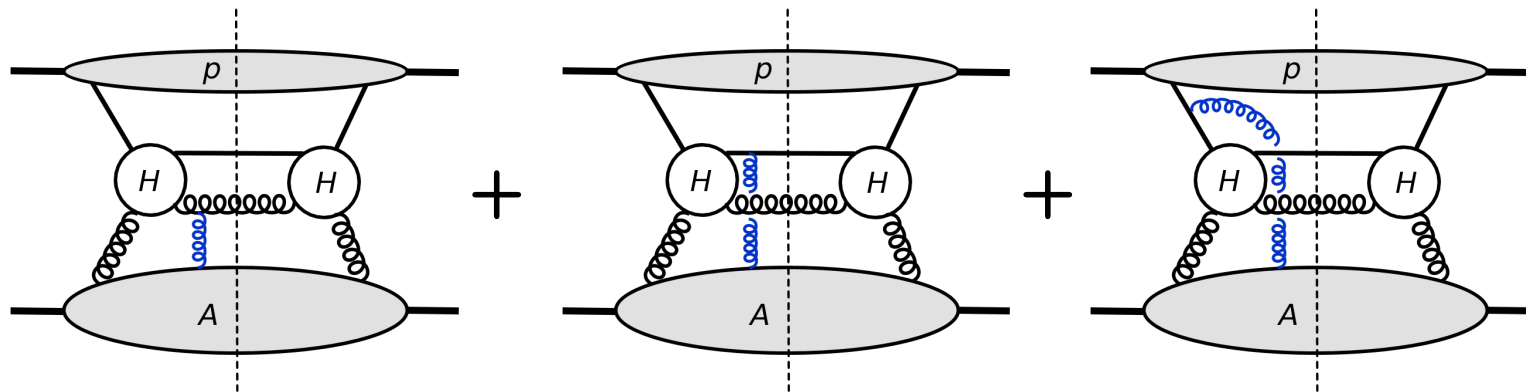
$$\mathcal{F}_{g/A}(x_2, k_t) \stackrel{\text{naive}}{=} 2 \int \frac{d\xi^+ d^2\xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) F^{i-}(0)] | A \rangle$$

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- a theoretically consistent definition requires to include more diagrams



+ similar diagrams with 2, 3, ... gluon exchanges

They all contribute at leading power and need to be resummed.

this is done by including gauge links in the operator definition

Process-dependent TMDs

- the proper operator definition(s) some gauge link $\mathcal{P} \exp \left[-ig \int_{\alpha}^{\beta} d\eta^{\mu} A^a(\eta) T^a \right]$

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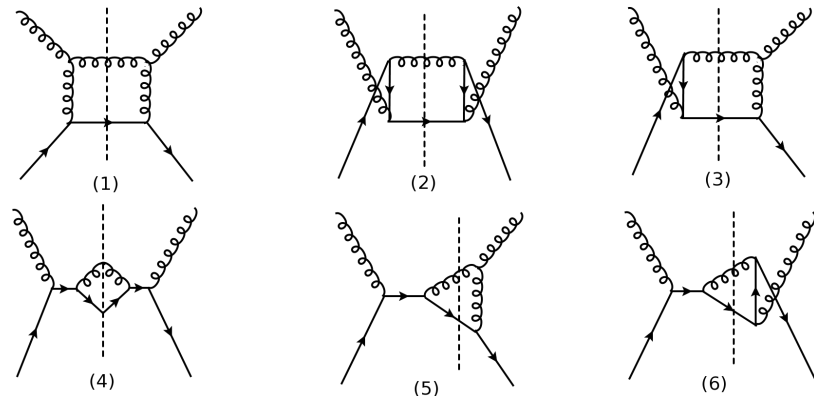
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- in general, several gluon distributions are needed already for a single process

example for the $qg \rightarrow qg$ channel

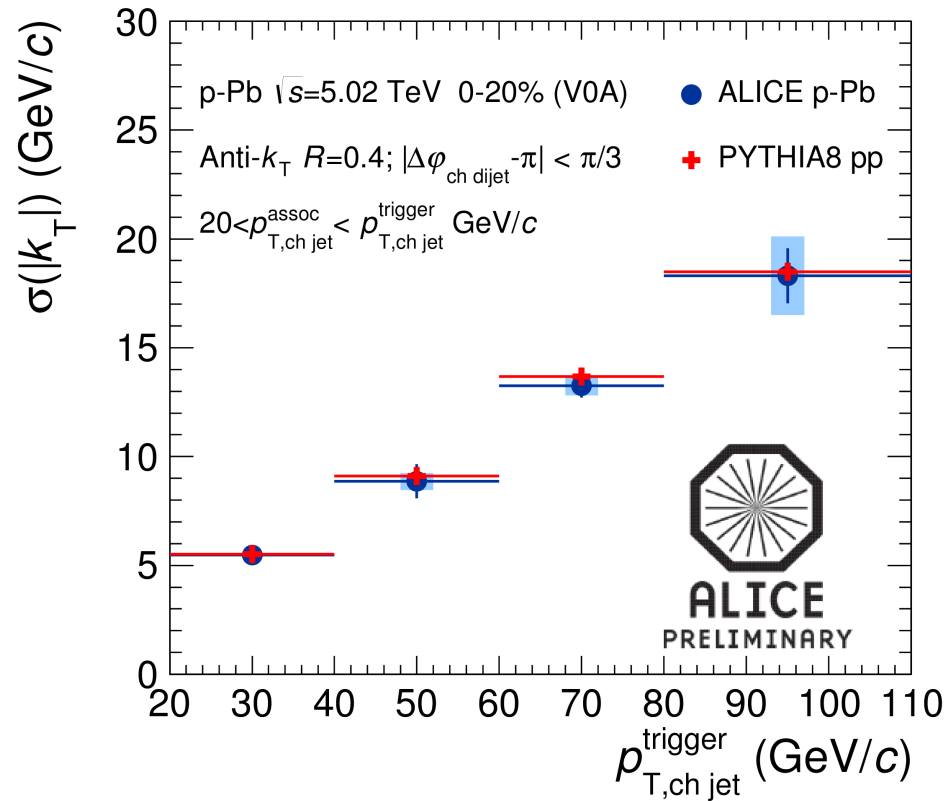
each diagram generates a different gluon distribution



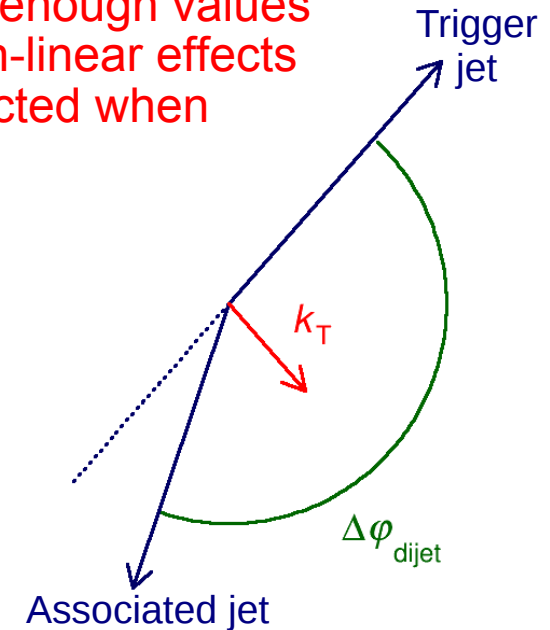
- the transverse imbalance of di-jets is sensitive to these fundamental features of QCD

Di-jet transverse imbalance

- no sign of nuclear effects at the LHC mid-rapidity

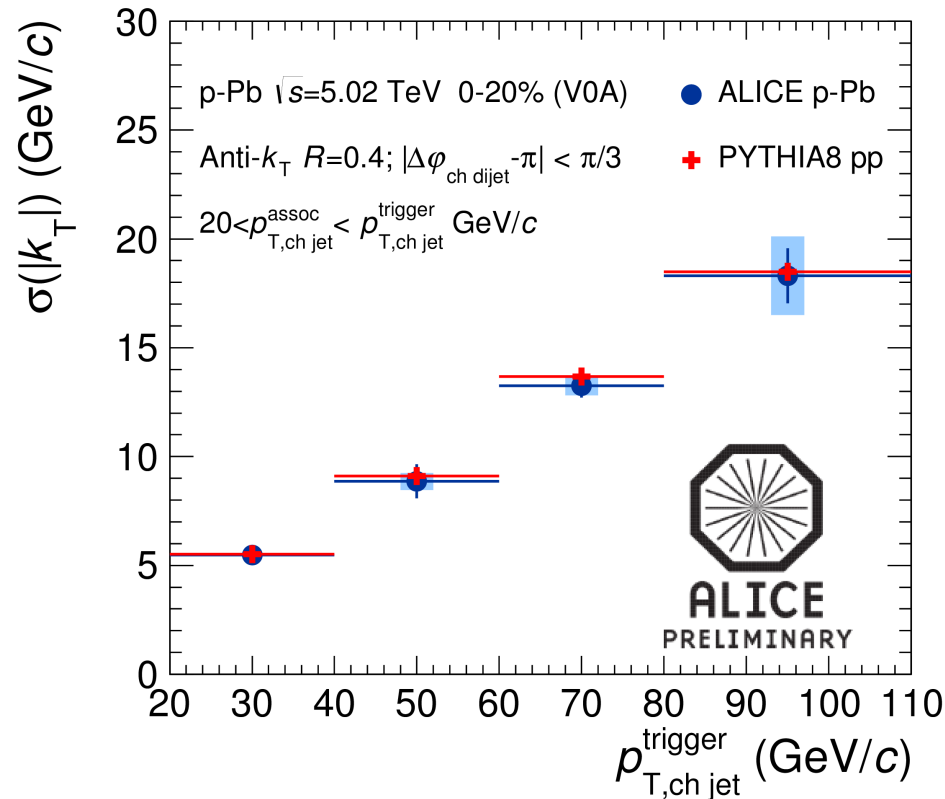


for small enough values
 of x_A , non-linear effects
 are expected when
 $k_T \sim Q_s$

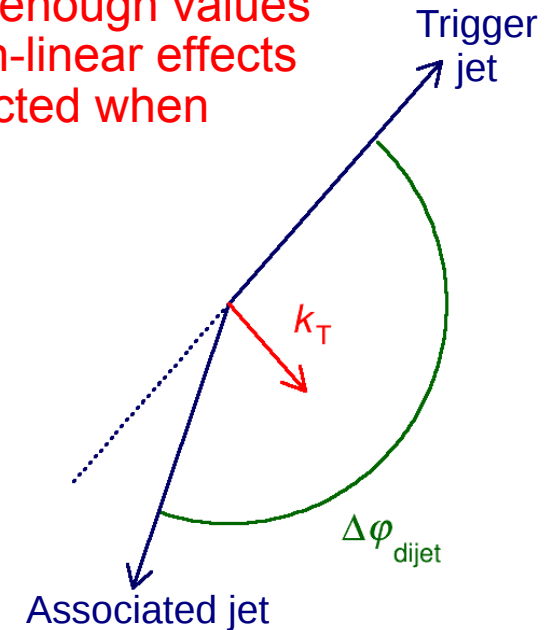


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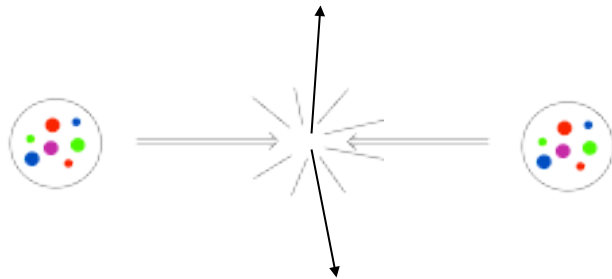
the di-jet imbalance is independent of A , and not related to Q_s
 all due to 3-jet final states, and perhaps some non-perturbative intrinsic k_T
 one needs to look at forward di-jet systems to see small- x nuclear effects

Di-hadron final-state kinematics

final state : k_1, y_1 k_2, y_2

$$x_p = \frac{k_1 e^{y_1} + k_2 e^{y_2}}{\sqrt{s}} \quad x_A = \frac{k_1 e^{-y_1} + k_2 e^{-y_2}}{\sqrt{s}}$$

scanning the wave functions:



$$x_p \sim x_A < 1$$

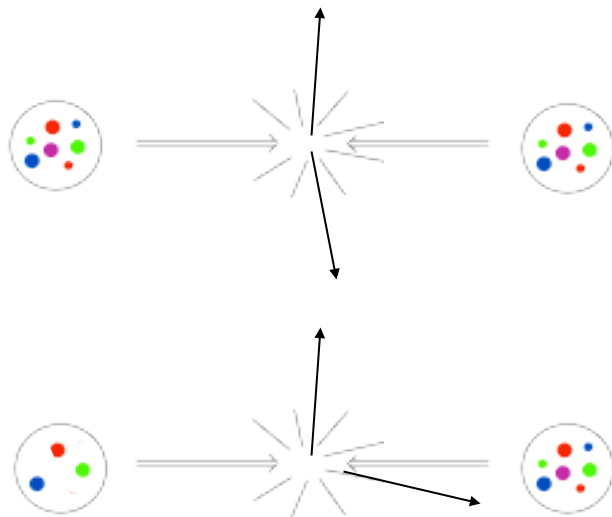
central rapidities probe moderate x

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$$x_p \text{ increases} \quad x_A \sim \text{unchanged}$$

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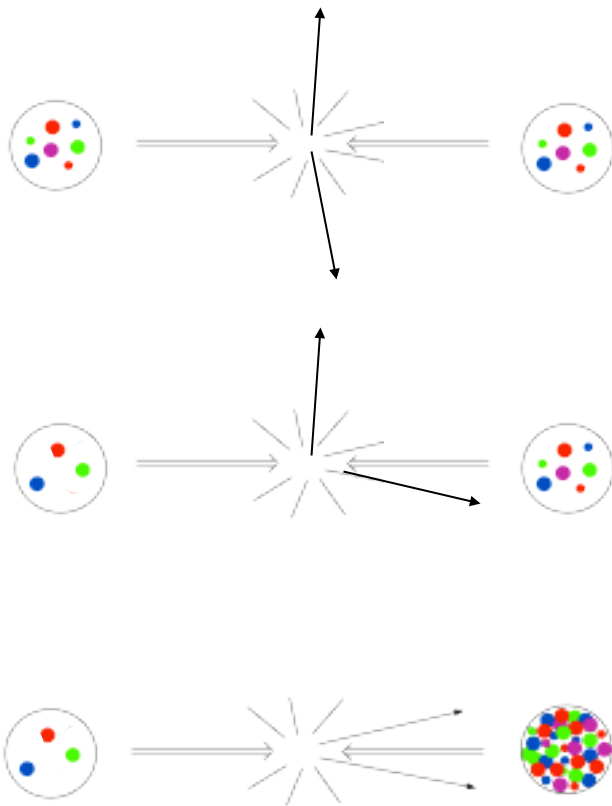
forward/central doesn't probe much smaller x

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$$x_p \sim \text{unchanged} \quad x_A \text{ decreases}$$

$$x_p \sim 1, x_A \ll 1$$

forward rapidities probe small x

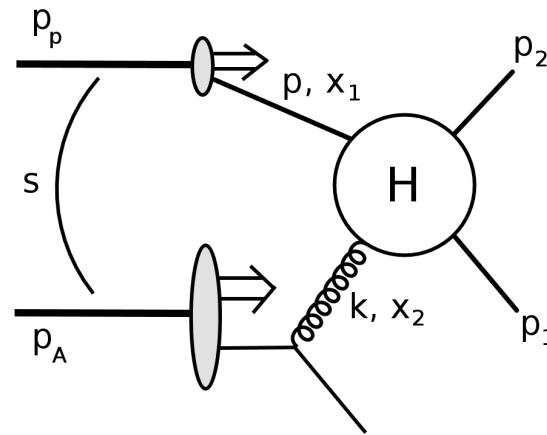
Theory of forward di-jet production in $p+A$ collisions

P. Kotko, K. Kutak, CM, E. Petreska, S. Sapeta and A. van Hameren

JHEP 1509 (2015) 106, arXiv:1503.03421

Dilute-dense kinematics

- large-x projectile (proton) on small-x target (proton or nucleus)



$$\hat{s} = (p + k)^2$$

$$\hat{t} = (p_2 - p)^2$$

$$\hat{u} = (p_1 - p)^2$$

Incoming partons' energy fractions:

$$x_1 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{y_1} + |p_{2t}|e^{y_2}) \quad \xrightarrow{y_1, y_2 \gg 0} \quad x_1 \sim 1$$

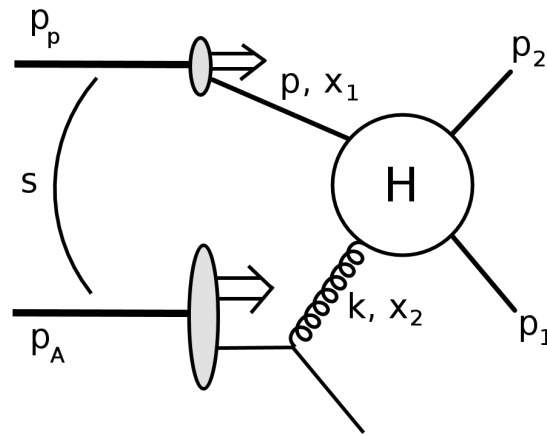
$$x_2 = \frac{1}{\sqrt{s}} (|p_{1t}|e^{-y_1} + |p_{2t}|e^{-y_2}) \quad \xrightarrow{y_1, y_2 \gg 0} \quad x_2 \ll 1$$

Gluon's transverse momentum (p_{1t} , p_{2t} imbalance):

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}|\cos\Delta\phi$$

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- several momentum scales in the process

$$|p_{1t}|, |p_{2t}| \gg Qs \quad \text{however, } |k_t| \text{ can be small or large}$$

Improved TMD factorization

valid for $|p_{1t}|, |p_{2t}| \gg Q_s$

but for an arbitrary value of the di-jet imbalance $|k_t|$

$$\frac{d\sigma^{pA \rightarrow \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1, \mu^2) \sum_{i=1}^2 K_{ag^* \rightarrow cd}^{(i)} \Phi_{ag \rightarrow cd}^{(i)} \frac{1}{1 + \delta_{cd}}$$

standard collinear pdf
for the large-x projectile

six unintegrated gluon distributions
for the small-x target (2 per channel)

and their associated hard matrix elements

$$\Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t^2)$$

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- improvements compared previous works

off-shellness of the small-x gluon crucial in the regime $|p_{1t}|, |p_{2t}|, |k_t| \gg Q_s$

several gluon TMDs (as opposed to a single one) crucial in the regime $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$ which corresponds to nearly back-to-back jets

The six 2-to-2 off-shell hard factors

- they can be computed in two independent ways:

using Feynman diagrams, and using color-ordered amplitudes

i	1	2
$K_{gg^* \rightarrow gg}^{(i)}$	$\frac{N_c}{C_F} \frac{(\bar{s}^4 + \bar{t}^4 + \bar{u}^4) (\bar{u}\hat{u} + \bar{t}\hat{t})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$	$-\frac{N_c}{2C_F} \frac{(\bar{s}^4 + \bar{t}^4 + \bar{u}^4) (\bar{u}\hat{u} + \bar{t}\hat{t} - \bar{s}\hat{s})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$
$K_{gg^* \rightarrow q\bar{q}}^{(i)}$	$\frac{1}{2N_c} \frac{(\bar{t}^2 + \bar{u}^2) (\bar{u}\hat{u} + \bar{t}\hat{t})}{\bar{s}\hat{s}\hat{t}\hat{u}}$	$\frac{1}{4N_c^2 C_F} \frac{(\bar{t}^2 + \bar{u}^2) (\bar{u}\hat{u} + \bar{t}\hat{t} - \bar{s}\hat{s})}{\bar{s}\hat{s}\hat{t}\hat{u}}$
$K_{qg^* \rightarrow qg}^{(i)}$	$-\frac{\bar{u} (\bar{s}^2 + \bar{u}^2)}{2\bar{t}\hat{t}\hat{s}} \left(1 + \frac{\bar{s}\hat{s} - \bar{t}\hat{t}}{N_c^2 \bar{u}\hat{u}} \right)$	$-\frac{C_F}{N_c} \frac{\bar{s} (\bar{s}^2 + \bar{u}^2)}{\bar{t}\hat{t}\hat{u}}$

$\hat{s}, \hat{t}, \hat{u}$ are the Mandelstam variables and $\bar{s}, \bar{t}, \bar{u} = \hat{s}, \hat{t}, \hat{u} (k_t = 0)$

The six TMD gluon distributions

- correspond to a different gauge-link structure

$$\mathcal{F}_{g/A}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) U_{[\xi, 0]} F^{i-}(0)] | A \rangle$$

several paths are possible for the gauge links 

- when integrated, they all coincide

$$\int^{\mu^2} d^2 k_t \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t^2) = x_2 f(x_2, \mu^2)$$

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- they are independent and in general they all should be extracted from data

only one of them has the probabilistic interpretation of the number density of gluons at small x_2

- their evolution with decreasing x can be computed in the Color Glass Condensate

The x evolution of the gluon TMDs

(in the simplest case)

- the Balitsky-Kovchegov (BK) evolution

Balitsky (1996), Kovchegov (1998)

$$\frac{d}{dY} f_Y(k) = \bar{\alpha} \int \frac{dk'^2}{k'^2} \left[\frac{k'^2 f_Y(k') - k^2 f_Y(k)}{|k^2 - k'^2|} + \frac{k^2 f_Y(k)}{\sqrt{4k'^4 + k^4}} \right] - \bar{\alpha} f_Y^2(k)$$

$$Y = \ln\left(\frac{1}{x}\right)$$

$$\bar{\alpha} = \frac{\alpha_s N_c}{\pi}$$

BFKL

non-linearity important
when the gluon density
becomes large

The x evolution of the gluon TMDs

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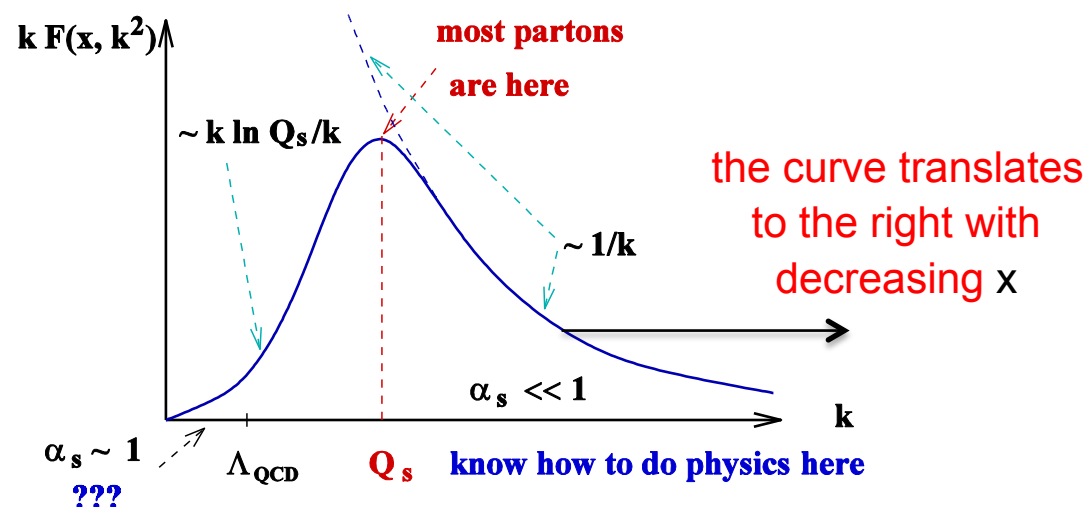
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$\underbrace{\hspace{15em}}_{\text{BFKL}}$

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non-linearity important when the gluon density becomes large

- solutions: qualitative behavior



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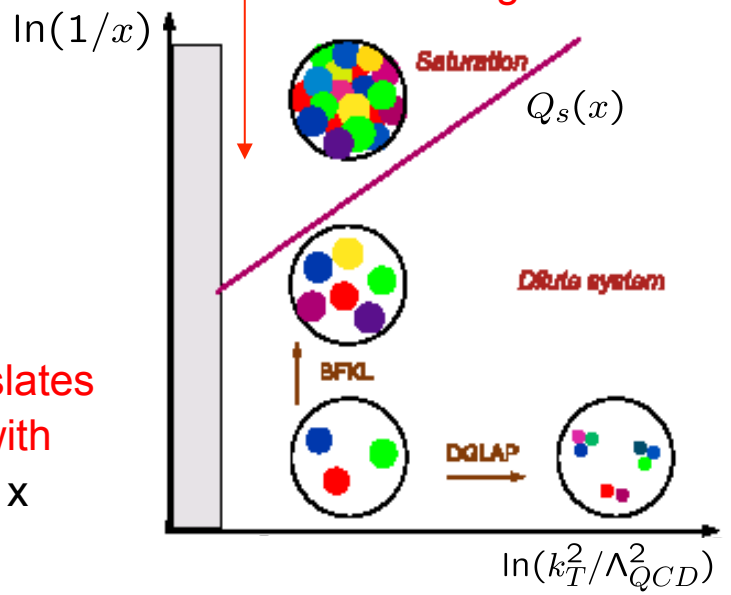
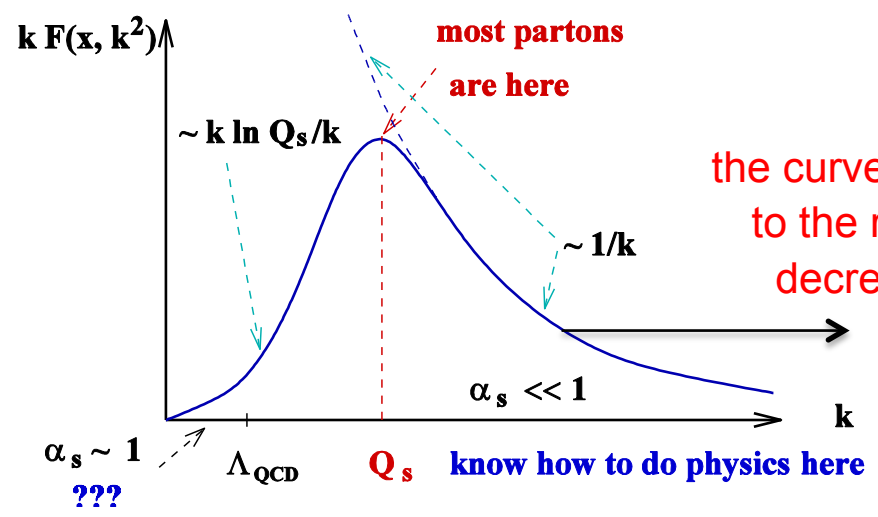
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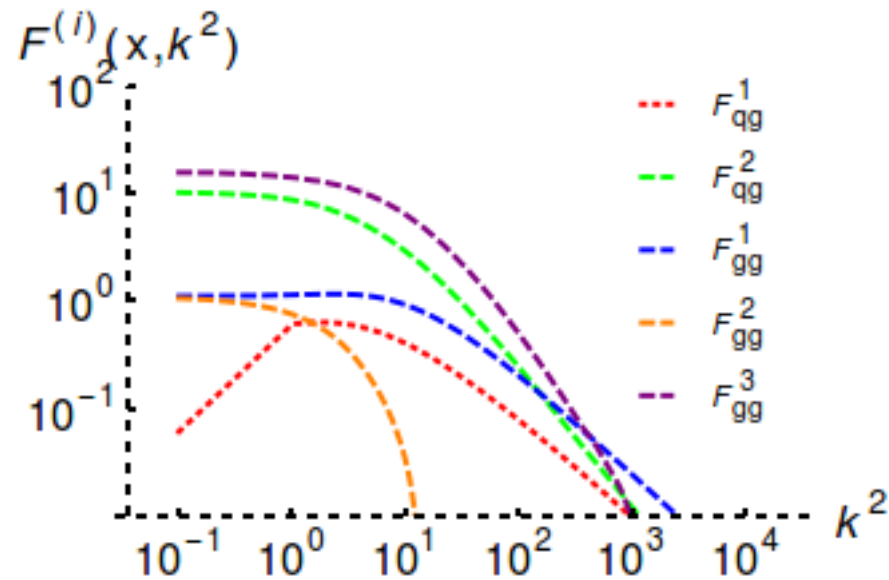
non-linearity important when the gluon density becomes large

- solutions: qualitative behavior



the distribution of partons as a function of x and k_T

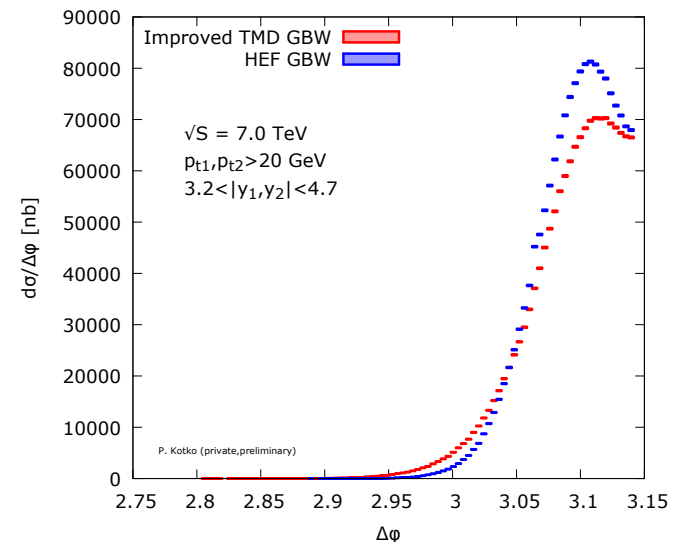
Some numerical results



the five gluon TMDs which survive in the large N_c limit

comparison with using only one gluon TMD (as done in several previous studies)

large differences for $\Delta\phi \simeq \pi$ as expected



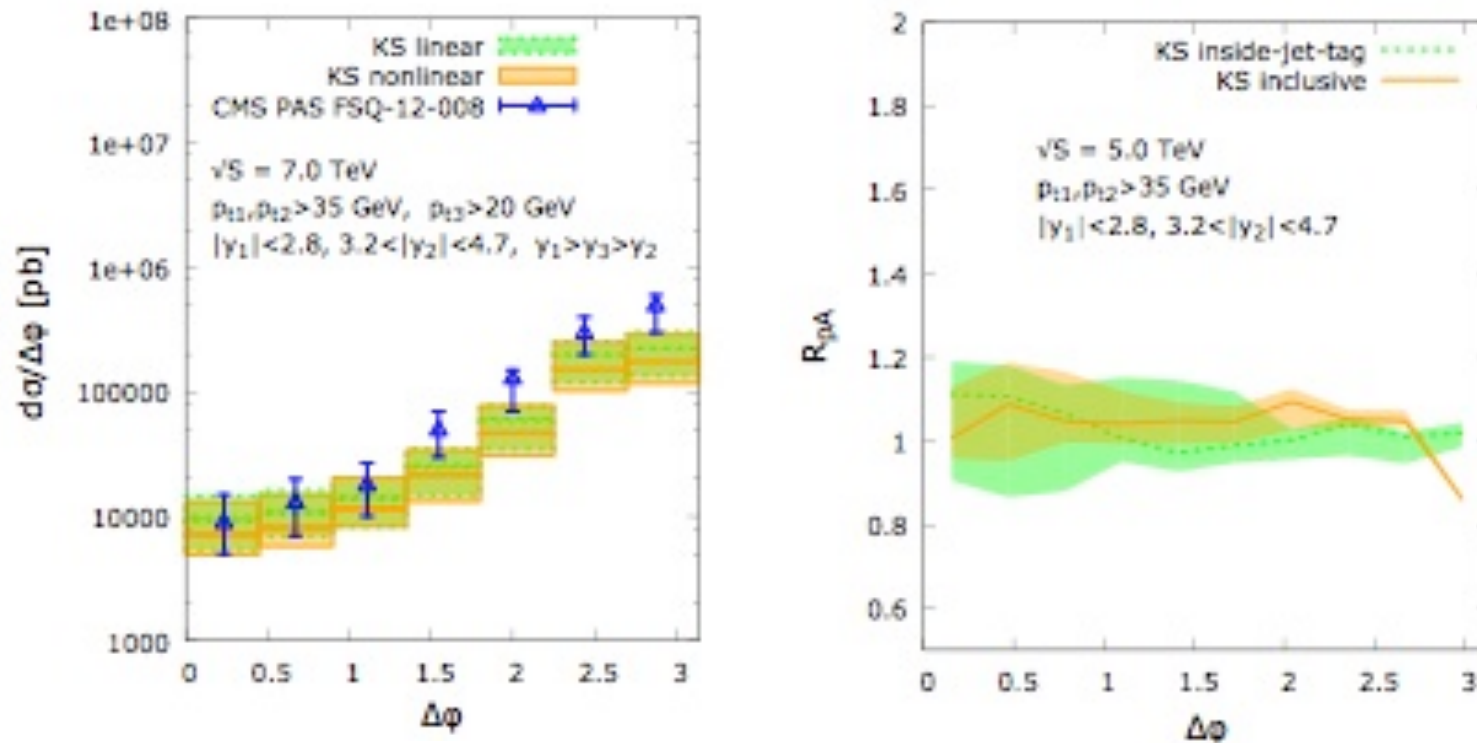
Phenomenology in p+Pb collisions at the LHC

A. van Hameren, P. Kotko, K. Kutak, CM and S. Sapeta

Phys.Rev. D89 (2014) 9, 094014, arXiv:1402.5065

CMS central-forward di-jet data

- non-linear effects are small, as expected



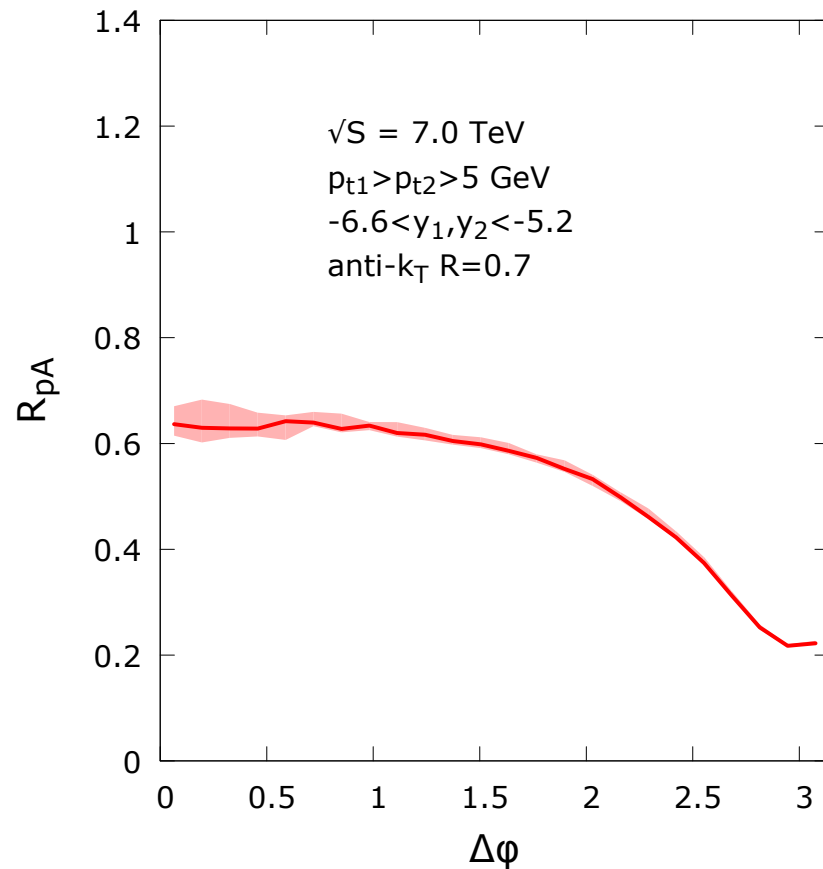
but this is a good test of the formalism,
which does a good job describing the data

van Hameren, Kotko, Kutak, and Sapeta (2014)

R_{pA} of forward-forward di-jets

- strong nuclear modification predicted

due to strong non-linear effects



the kinematics for this plot are chosen assuming that the di-jets are detected in an upgrade of the CASTOR detector

for detection in the central part of CMS (i.e. $3 < y < 5$), R_{pA} goes from 1 to 0.6 with increasing $\Delta\phi$

- prospects for a measurement

CMS: analysis started but now on hold

ATLAS: expressed interest to do it

ALICE: forward jets impossible?

LHC-b: forward jets possible?

Conclusions

- Nuclear modifications of di-jet properties in p+Pb already studied at mid rapidity:
 - small effect on the longitudinal distribution explained by nPDF framework
 - no effect on the transverse structure as expected in the nPDF framework
- Our goal: study the nuclear effects on the transverse imbalance expected at small-x, i.e. at forward rapidities
 - at small-x, TMD nuclear gluon distributions become relevant
 - we calculated them using the CGC framework
 - we predict a suppression of the forward-forward di-jet R_{pA}
 - now hoping for experimental data in p+Pb
- Several theory improvements still needed:
 - correct treatment of nuclear impact-parameter dependence
 - estimate effects of jet fragmentation